FEDERAL ESTIMATES OF POPULATION AND FEDERAL SUPERVISION OF LOCAL CENSUSES.

INTRODUCTION.

A census of population is taken decennially by the Census Bureau, but in the preparation of its annual and periodic reports there is constant need for statements of population covering the years between the decennial enumerations. To meet this need, estimates of population for states and cities are used, based upon the so-called arithmetical method.

Ten or more states are accustomed to take a census midway between the Federal enumerations, but it has been deemed inadvisable by the Census Bureau to use the results of these state enumerations in computing estimates of population for the states and cities concerned.

A similar course has been pursued with respect to the results of police and other local censuses, but means are provided for securing, upon request, an authoritative census of population for a city or village, or even a state, under the official supervision of the Census Bureau.

Inasmuch as there is frequent inquiry regarding the practice of the Bureau of the Census of basing its estimates upon the results of Federal censuses only, without taking into account the results of state, police, or other local censuses unless such censuses have been taken under Federal supervision; and inasmuch as there are also numerous requests for a statement of the course to be pursued in securing the taking of a local census under the supervision and control of a representative of the Census Bureau, this circular has been prepared for the purpose of furnishing information on these points.

TEDERAL CENSUSES.

The Bureau of the Census is required by law to take a census of the United States once in every 10 years. The last Federal census was taken as of April 15, 1910, and constituted the thirteenth decennial enumeration of population since the adoption of the Constitution.

No provision thus far has been made for a more frequent enumeration of population, although since the organization of the permanent Census Bureau in 1902 the need for an intermediate or quinquennial

(3)

census has been increasingly felt, particularly as a basis for computing per capita rates in connection with the statistics of deaths and financial statistics of cities, both of which are now compiled annually.

Prior to the Twelfth Census the entire work of each decennial census had related to the census year, although it had successively grown to include many subjects of investigation in addition to the mere enumeration of population originally provided for; and the work was carried on at each decennial period under a purely temporary organization brought together expressly for the purpose and entirely disbanded a few years later, even though the work, as originally outlined, had not been wholly completed or all reports published.

Under the law governing the taking of the census of 1900, the work to be undertaken during the decennial census period (July 1, 1899, to June 30, 1902) was restricted to inquiries relating to four subjects only—population, agriculture, manufactures, and mortality—and was to be completed not later than July 1, 1902; but authority was given to take up certain special subjects of investigation after the work relating to the general subjects just named had been completed.

The decennial census of 1910 was confined to the subjects of population, agriculture, manufactures, and mines and quarries, provision having been made meanwhile for the annual collection of data relating to mortality and certain other subjects. The collection of statistics of manufactures and of mines and quarries, however, was made by special agents appointed solely for this work, so that the only duties of the enumerators were to obtain statistics of population and agriculture.

At each decennial enumeration of the population since 1880 an increasingly large number of supervisors of census has been provided for by law, and every practicable aid has been given them by the officials of the Census Bureau to insure painstaking and accurate work on the part of the census enumerators. The law under which the census of 1900 was taken provided 300 supervisors, but for the census of 1910 the number was increased to 330. These supervisors of census were allowed clerk hire and, in addition, the services of a certain number of special agents, according to the varying demands of the work, to assist them in overseeing the work of the enumerators, of whom in 1910 there were 71,580, as against 53,321 in 1900. A test examination was necessary for the appointment of the census enumerators, and they were given, in most cases, oral instruction as to their duties, in addition to the book of printed instructions furnished for their guidance; the actual work of enumeration, so far as practicable, was closely supervised as it progressed daily; interpreters were employed wherever necessary; and the completed returns were given careful and, to an extent, critical examination before the enumerators' accounts were finally passed for payment. In the enumeration of the population of cities, also, special means were provided for checking the work of the enumerators, and special blanks were used for the enumeration of the boarding-house and lodging-house population and for securing the required information concerning families absent from their usual places of residence at the time the census was taken. In short, every reasonable means was taken by the Federal census officials to secure a full and accurate enumeration of population in each and every section of the country.

ESTIMATES OF POPULATION.

The organization of the permanent Census Bureau in 1902 and its prosecution thereafter of the various special subjects of investigation designated by the law creating it, raised the question immediately as to the wisdom of making estimates of population for the years following a decennial census, in order to make the information thus gathered by the Census Bureau more valuable. It was apparent that the statistics obtained by the special reports prepared in the intervals between decennial censuses—as required by the permanent census act and by later legislation—would not be comparable with the statistics of population as reported at the last preceding decennial census, and that their statistical value would be greatly impaired so long as comparison with the population was impossible.

The matter was given most careful consideration at the time, by a committee of experts in the Census Bureau and by correspondence with leading experts throughout the country, and it was finally decided to compute estimates of population but to confine them as closely as possible to those areas for which figures were required. Since the only purpose for which these estimates were to be prepared was to provide a basis for the computation of statistical rates, it was felt that, even though in individual cases they would be undoubtedly too low, or, indeed, occasionally too high, they would in general serve a useful purpose in making the statistics significant, and even with a considerable margin of error in the estimated population would give approximate rates, such as death rates, rates of taxation, per capita, and the like.

The first estimates of population made in accordance with this decision were published in bulletin form in April, 1904, and gave the estimated population as of June 1 for the years 1901, 1902, and 1903. The estimates contained in this bulletin, although subjected to some criticism, as was to be expected, were in general well received, and there has been a constant demand for them and for similar estimates for later years, especially by registrars of vital statistics. A second bulletin was accordingly issued in March, 1907, containing

estimates of population as of June 1 for the years 1904, 1905, and 1906, and a third bulletin in March, 1914, containing estimates as of July 1 for 1910, 1911, 1912, 1913, and 1914. A fourth bulletin, containing estimates of population for the years 1910 to 1916, inclusive, is to be issued in the near future. Estimates of population for the years between 1906 and 1910 were also computed by the Census Bureau for use in the preparation of its annual and other reports, but these estimates were not separately published in bulletin form.

METHOD OF ESTIMATING POPULATION.

In computing these estimates of population, the so-called arithmetical method has been used, as being at once the simplest and as having been shown by experience to result in a closer approach to accuracy in the majority of cases than any other available method. It rests on the assumption that the annual increase for each year since the last census would be one-tenth of the decennial increase between the last two censuses. In making these estimates, cognizance was taken of changes due to the annexation or detachment of territory, and where for areas thus annexed or detached the exact population at the last preceding enumeration could not be determined from the census figures the approximate population was ascertained, so far as possible, by correspondence with local officials.

This method was adopted by the Federal Census Bureau as being, on the whole, the best for the purpose and as producing estimates which, in general, would be within 6 per cent of the truth. At the time when the question of making estimates of population was under consideration by the census officials a most careful study was made of four other methods, namely, the basing of population estimates (1) upon the number of votes cast, (2) upon a directory canvass, (3) upon a school census, and (4) upon an assumed constant rate or percentage of growth between decennial censuses. It was found from actual tests, however, that none of these alternative methods produced results which approximated the true population as closely as the simple method adopted by the Federal Census Bureau. The main conclusions of the study regarding the accuracy of the several methods tested are summarized as follows:

1. Number of votes cast.—In making estimates of population by this method the usual process is to multiply the number of votes cast at the last preceding election by the presumed ratio between the number of inhabitants and the number of votes, for example, 4, 5, or 6. The number of votes cast is variously affected, however, not only by the importance of the election, as a presidential election compared with a state or local election, but also by the ratio of males to total population, the ratio of adult males to all males, the ratio of

citizens to all male adults, the ratio of legal voters to citizens, and the ratio of actual voters to legal voters, so that there can be no constant or uniform relation of votes cast to population. In three northern cities there were fewer than four inhabitants to each vote cast at the presidential election of 1900, while in several other northern cities there were more than eight and in certain southern cities more than twelve to each vote cast, and the general results obtained by this method indicated a probable error of more than 12 per cent.

- 2. School census.—Estimates of population are sometimes obtained by multiplying the number of children of school age, as shown by a school census, by the ratio which the total population is thought to bear to the number of persons of this age class. The number of children of school age, if correctly ascertained, is a fairly accurate index to the total population and will yield results half of which fall within 5 or 6 per cent of the truth. But the number of children of school age is so seldom ascertained with accuracy by a school census that this method is found of little practical value.
 - 3. A directory canvass.—Estimates of population derived from this source are made by multiplying the number of names in a directory by a ratio assumed to represent the most probable number in the total population to each name in the directory, even though it may include names of persons residing outside but adjacent to the city limits. The evidence shows, however, that the ratio between the number of names in a directory and the total population is almost unknown, but varies with the age and sex composition of the population and the conditions and rules of the directory canvass; and that estimates so based result uniformly in too large a figure and usually in very serious inaccuracy, representing a probable error of more than 10 per cent.
 - 4. Assumption of a constant rate of growth.—This method of estimating population rests upon the assumption that the percentage of increase remains constant from one decade to another, but this assumption is not borne out when the test is applied to American cities. An examination of the rate of growth in each of 78 cities between 1890 and 1900, as compared with that between 1880 and 1890, indicated a difference of 18 per cent, thus showing that the assumption that under present conditions the rate of growth of a given city tends to remain the same is inadmissible.

A test applied at the same time to the method which was finally adopted showed that between 1880 and 1890 the population of 78 cities together increased by an average annual amount of 407,028, and between 1890 and 1900 by an average annual amount of 414,793. As the rate of increase for the cities collectively fell from 46.8 per cent between 1880 and 1890 to 32.5 per cent between 1890 and 1900,

while the total amount of increase between 1890 and 1900 exceeded by less than 2 per cent the total amount between 1880 and 1890, it is evident that the assumption of a constant amount gives results much nearer the truth than the assumption of a constant rate. If this method had been applied to the cities for 1900 the results in half the cases would have been within 6 per cent of the truth—closer than estimates based upon the assumption of a constant rate of increase or upon votes cast or numbers of names in directories, and as close as estimates based on an accurate census of school children, were that obtainable.

STATE CENSUSES.

In the preparation of the estimates of population for certain states for the years 1904, 1905, 1906, 1907, 1908, and 1909, for the official use of the Census Bureau, the population figures as ascertained by state censuses taken in or about 1905 were used. After the Federal census of 1910 was taken and it became possible to compare the results of that census with those shown by the state censuses of 1905, it was decided to abandon the use of state census figures and thereafter in computing estimates to use the population figures as returned at Federal censuses only. The estimates of population for the years 1910, 1911, 1912, 1913, and 1914, made as of July 1 in each year, were therefore based upon the results of the Federal enumerations of 1900 and 1910. This decision affected the estimates for 14 states only, so far as the use of state census figures for 1905 was concerned, and would have applied to only 10 states from and after 1915, 4 states having failed to take censuses in or about that year, although each had been long accustomed to take a census in the intercensal period.

WHY STATE CENSUSES CAN NOT BE ACCEPTED.

The principal and governing reason why it has been decided that the population shown by state censuses can not be accepted and used by the Bureau of the Census as a basis for the Federal estimates of population is that these censuses are entirely beyond the control of the Census Bureau and it, therefore, has no knowledge, as of itself, of the methods pursued in taking them or as to the care and accuracy with which the work of enumeration is carried on. There is uncertainty even in regard to what the population reported by a local census includes or is supposed to include; that is, whether it includes all persons present in the particular city or village on the day the

¹ A state census was taken in 1904 by one state, Michigan, and in 1905 by 13 states, namely, Florida, Iowa, Kansas, Massachusetts, Minnesota, New Jersey, New York, North Dakota, Oregon, Rhode Island, South Dakota, Wisconsin, and Wyoming. The four states which failed to take censuses in or about 1915 were Michigan, Minnesota, Oregon, and Wisconsin.

census was taken, whether residents or not, or only those who actually reside there, whether present or temporarily absent from home at the time. The provisions of law under which these state censuses are taken and the practices followed in making the enumeration vary from state to state. The returns are in certain cases made up by local assessors, incidentally to the performance of other duties, and are admittedly more or less incomplete or inaccurate. This is particularly likely to be true where the assessor is required to determine Whether or not a person is liable to pay a poll tax and as a conse-Quence he is likely to be dodged by some or many persons; and to the extent that this is possible the census suffers and the results of an enumeration of population so conducted can not be altogether satisfactory. In some states, however, enumerators are appointed, and the census report is detailed, undoubtedly accurate, and of great Furthermore, there is much irregularity in the date as of Which the census is taken in the several states, representing in 1915, for the 10 states which then took a census, as many as seven different dates, as follows: January 1; March 1 (2 states); April 1 (2 states); first Monday in April; May 1; June 1 (2 states); July 1. State cen-Suses are also taken in some cases at more or less irregular intervals. For example, Oklahoma, under the territorial law which provided for a biennial census, took a census in 1908, following the Federal enumeration taken by order of the President on the occasion of its admission with Indian Territory as a single state in 1907, and has taken its census biennially since that date. In addition, it appears, as before stated, that four states which took censuses in or about 1905 did not take similar censuses in 1915, partly because of the large expenditure involved or of the inadequacy of the results with respect to the amount of money expended for the purpose; and since the taking of the state censuses in 1915 other states have canvassed the advisability of repealing the provisions for taking such censuses and relying upon the Federal Census Bureau for exact statements or, in lieu thereof, estimates of population to meet state or local needs.

The question raised by the department of health for the city of New York as to the sufficiency of the recent New York state census, so far as the population of New York City is concerned, may well be cited as an illustration of the reason why the Federal Census Bureau must maintain the position which it has taken with respect to the omission, as a part of the basis of its official estimates, of all statements of population which do not rest upon enumerations taken under its own jurisdiction or upon those which have been officially supervised by one of its representatives.

New York, under a constitutional provision, has taken a state census every 10 years, with two exceptions, since and including 1825.

No census was taken in 1885 or in 1895, but one was taken under legislative authority in 1892. The census provided for in the constitution was again taken in 1905, when, at the request of the secretary of state of New York, the work was supervised by an experienced official of the Federal Census Bureau; and it was again taken in 1915, under the immediate supervision of the secretary of state. In the law governing the taking of the census of 1915 provision was made for the appointment of enumerators, enumeration supervisors, and, for certain counties, chief enumeration supervisors; the inquiries to be made concerning each person enumerated were in considerable detail and quite sufficient to identify fully and determine the status of each individual with respect to his residence, relationship to the family in which he resided, and condition of citizenship or alienage, as the state constitution requires; and each and every person, under the terms of the census law and the printed instructions, was to be enumerated at his or her usual place of abode, in much the same manner apparently as the Federal census law provides.

The results of the enumeration for New York City, and particularly with respect to Manhattan Borough, were not satisfactory to the health department of that city, and the figures as announced by the secretary of state were the subject of a vigorous protest on their part, in the discussion of which the state health department and the state census supervisor participated and to which much publicity was given. The reasons for the protest by the city health department were, in brief, as follows:

- (1) That the instructions to the enumerators were to count only *permanent* residents, the word "permanent" being variously interpreted by the enumerators.
- (2) That the dwellers on the nine Government reservations, military and naval, in the city, were wholly excluded, according to the statement of the state census supervisor, following an opinion obtained from the attorney general of the state.
- (3) That the summer exodus of many city dwellers was complete when the enumeration was made in June, 1915, and that a great many houses and apartments were boarded up and closed until October, with the result that the occupants were not counted at all.
- (4) That the complete returns of the state census authorities show that the population of the city of New York on June 1, 1915, was 5,047,221, but that an estimate based upon the state census of 1905 and the Federal census of 1910, made according to the so-called arithmetical method used by the Federal Census Bureau, which method has been considered ultraconservative as applied to New York City, showed a population as of July 1, 1915, of 5,585,772, an excess of 538,551.¹

¹The estimate of population for New York City, based upon the population as returned at the Federal censuses of 1900 and 1910, made as of July 1, 1915, was 5,468,190, and as of June 1, 1915, the date when the state census was taken, was 5,456,970.

- (5) That, if use is made of the check on the estimate of population often used by vital statisticians, by assuming that the birth rate as it actually was in the year of the census taking (in this case in the year 1910) was the same in 1915 (26.87 per 1,000 of the population) and applying it to the number of births actually reported in the year 1915 (141,356), the resulting figure will be in excess by more than 200,000 of the population of the city as furnished by the state enumeration,
- (6) That, if it is assumed that the birth rate has declined since 1910 (a decline which is known to have been universal in all civilized countries during the past 10 years, and to which rule of general decline New York City is no exception)—one point for 1,000 of the population—the resulting estimate of the population would be in accord with the estimate for 1915 (see (4), p. 10) made by the arithmetical method used by the Federal Census Bureau.

(7) That, according to the returns of the state census, the city grew i, population between the years 1910 and 1915 at the rate of 5.9 per cent, while the population of the remainder of the state grew at the rate of 6.3 per cent; and that this is unprecedented in the statistical history of the state in the past and is a complete reversal of all past experiences.

The Federal Census Bureau, although repeatedly invited, consistently refrained from becoming a party to the discussion in any way, and is not in a position to judge as to the merits of the issue raised. In the case of state censuses or other enumerations made in the intervals between the Federal decennial censuses, it must be manifest that the Federal Census Bureau has no adequate means at hand for judging as to the methods employed or as to the accuracy of the results attained, and also that, after the completion of such a census. it would be entirely impracticable for it to undertake an investigation of sufficient magnitude and thoroughness to warrant a conclusion on its part as to the validity of a census taken by state or city authority at its own expense and for its own purposes. Without such an investigation, however, the Census Bureau could not, in justice, decide to accept one census and reject another from ex parte statements only; its decision, if adverse, would certainly be resisted, and so the bureau is forced to adhere to the policy, adopted after most careful consideration, of using for its own official purposes such figures only as are derived from Federal enumerations or from those state or local enumerations in which it has participated or over which it has had official supervision and control.

POLICE AND OTHER LOCAL CENSUSES.

Police censuses, and local censuses generally, are taken under a municipal ordinance or order and are made without much preparation, and in most cases probably do not require much preparation. They are undertaken principally to ascertain the number of inhabitants in a city, village, or other municipality at a given time, and oftentimes do not go beyond an inquiry as to the number of persons in the house, building, or other place in which persons are found or

in which they may sleep. The statements given as to the number of occupants are not always or often questioned, nor is particular inquiry always made as to the right of any or all of them to be included as a part of the population of the city or village being canvassed. The stimulus of local pride or desire is apt to result in excessive figures of population, and it is obvious that the results of such enumerations can not be given due credence without investigation and nearly always some curtailment of population. The District of Columbia, for example, has been accustomed to take a police census, usually every two years, but the Federal Census Bureau has not made use of the results of these enumerations, because they have not been made according to Federal census requirements and because they have included a very considerable population present in the District of Columbia at the time because of the sessions of Congress or for other reasons, but without actual residence there. The most recent of these censuses, however, appears to have been taken more nearly in conformity with census methods, and the results are more nearly in accord with the probable facts of population, partly, at least, because it was taken at a time when Congress was not in session.

As in the case of the state censuses, so it is with respect to police or other local enumerations. The results of these purely local censuses, as ordinarily taken, are not available for the purposes of the Federal Census Bureau, and can not be made available through any practicable system of investigation after the census has been taken. The only means by which their substantial accuracy can be assured is the adoption of proper measures beforehand, so as to provide for the prosecution of the actual work of enumeration in accordance with the methods employed by the Federal Census Bureau.

LOCAL CENSUSES OFFICIALLY SUPERVISED.

A system of Federal supervision and control of local censuses is entirely feasible and, indeed, has recently been successfully employed in the enumeration of the population of several cities and villages in different sections of the country. These special censuses were taken at the request of the municipal authorities, because it appeared that the city or village in question had grown so rapidly in population since the date of the last Federal census that the estimates of the Federal Census Bureau would not approximate in any sense its true population at the present time and because it was believed that it would be greatly to the advantage of the municipality to have a correct enumeration of the present population authoritatively made and published; they were taken wholly at the expense of the city or village, but were officially supervised and controlled by

a representative of the Federal Census Bureau; the enumeration was made, in each case, by order of the President of the United States, under the legislative authority of section 8 of the act creating the Department of Commerce and Labor (from which the Department of Commerce was later formed); and most satisfactory results have been reached through this cooperative plan of work.

PROCEDURE WITH RESPECT TO LOCAL CENSUSES.

A considerable number of other requests for similar special censuses have been received and are now awaiting action, and it is probable that still other requests will be made as time goes on. As a matter of general information, therefore, the method of procedure with respect to the special censuses thus far completed is briefly explained, as follows:

1. In response to the preliminary request for or inquiry regarding the possibility of securing a special census an estimate as to the probable cost was made and furnished, and the writer informed that before any action could be taken in the matter a request must be made by an official or representative of the city or village to the President of the United States, setting forth the reasons for the taking of such a special census.

2. Acting upon these representations, and under authority of section 8 of the organic act creating the Department of Commerce and Labor, which authorizes the President to make such investigations from time to time as he may deem proper, the President directed the Secretary of Commerce to in-

struct the Director of the Census to make the desired enumeration.

3. The Director of the Census, with the approval of the Secretary of Commerce, then issued a general order fixing the date as of which the special census was to be taken, designating the representative of the Federal Census Bureau who was to supervise the enumeration, and specifying the conditions under which the work was to be done.

4. An agreement was then entered into between the Bureau of the Census and the proper official or representative of the city or village, under the terms of which there was deposited in a bank a sum sufficient to cover the estimated cost of the special census, in the name of the representative designated by the Director of the Census to supervise the work. This was to be available (a) for defraying all expenses for railroad fare, hotel bills, street-car fares, and bus fares, and for all other reasonable and necessary expenses incurred by the supervisor in connection with the work of the enumeration; and (b) for the compensation (not to exceed 2 cents per name) of the enumerators employed by the supervisor to carry on the work, and the payment of all necessary expenses incurred by them. It was further provided that the unexpended balance, if any, remaining in the bank after the payment of the above-mentioned expenses should be returned to the city or village. It was also stipulated in the agreement that the city or village should, at its own expense, provide suitable office room, furniture, and supplies for carrying on the work, and printed schedules for the use of the enumerators, in accordance with a form to be prepared by the supervisor. The agreement also provided that the representative of the Census Bureau should have full and complete charge of every feature of the work, and should not be interfered with by any representative of the

city or village; that each enumerator should be made an employee of the Census Bureau and be subject to the Federal laws governing census taking; that the data collected by the enumerators should be regarded as confidential; and that the Bureau of the Census should tabulate these data and announce the results at as early a date as practicable.

COST OF LOCAL CENSUSES.

The total and per capita cost of each of the special censuses thus far taken, including both the cost of supervision and clerical assistance and of the enumeration itself, the number of enumerators employed, and the total population thus enumerated, were as follows:

CITY OR VILLAGE.	Num- ber of enu- mera- tors,	Total cost.	Cost per capita (cents).	Popula- tion enumer- ated.	POPULATION IN-	
					1910	1900
Tulsa, Okla Hamtramck, Mich Highland Park, Mich St. Clafr Heights, Mich Hastings, Nebr El Paso, Tex	15 12 6 13	\$837. 66 691. 69 828. 06 256. 36 548. 42 2,059. 37	2.97 3.21 3.05 3.74 5.04	28, 240 21, 520 27, 170 6, 853 10, 874 4 61, 898	18, 182 3, 559 4, 120 1, 252 9, 338 30, 279	11,390 (2) 427 (2) 7,188 15,906

¹ Population according to special census of 1907, 7,298.

² Not incorporated in 1900.

³ Based on total number of persons enumerated within the city limits (70,711).

According to these figures, the average per capita cost of these special censuses was about 3\frac{1}{3} cents, although in one case, owing to exceptional causes, it was slightly over 5 cents.

The form of schedule used was made as simple as possible, and the inquiries made regarding each person enumerated covered six items only, namely, residence (street and house number), name, relationship to head of family, sex, color or race, and age at last birthday.

DESIRABILITY OF INTERCENSAL ENUMERATION.

There should be, as a matter of fact, a more frequent general enumeration of population by the Federal Government, and an intercensal enumeration on a limited scale could well be made at a minimum expense as compared with the cost of the much more elaborate census which has come to be taken decennially and which necessarily must cover many topics of inquiry with respect to the population as well as substantially complete statistics as to agriculture. Such an intercensal enumeration need not cover much more ground than the special censuses recently taken, except that an inquiry should be added to cover the country of birth.

Not including 7,051 Mexican refugees and 1,762 soldiers temporarily residing in El Paso.

A decennial census obviously does not meet the requirements of present-day activities, but a more frequent enumeration of the population of the country can not be had until the necessary authority is provided by Congress. In the meanwhile the Bureau of the Census, for want of actual figures, must resort to estimates for use in its own investigations and reports; and it periodically prepares and publishes these estimates, with proper explanations as to the method of compilation and their limitations in point of accuracy. But the bureau, so far as its work in other directions will permit, is prepared to supervise local or special censuses, upon the request and at the expense of the local authorities, and will then accept the results of such censuses and modify its estimates accordingly.

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